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ARE THERE LIMITS TO GROWTH?*

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A simple theoretical model of pollution is developed that generates an inverted U-shape relationship between per capita income and environmental quality. This model is then used to study long-run growth. The same inverted U-shape is shown to appear in time series, and the prospects for sustained growth are shown to hinge on whether increasingly strict environmental regulation is compatible with a constant rate of return on capital. Implementation is also studied. Tax and voucher schemes are shown to have an advantage over direct regulation because they provide the correct incentives for capital accumulation.

1. INTRODUCTION

Will pollution increase unabated as current LDCs develop? Newspaper accounts of factory smokestacks spewing out soot in Mexico, Russia, Eastern Europe, China, and other rapidly growing economies paint a grim picture. But London in the late nineteenth century and Pittsburgh in the early twentieth were probably not so different from Mexico City and Krakow today. Substantial evidence points to an inverted U-shape relationship between per capita income and various types of pollution, suggesting that while the early stages of economic growth cause the problem, later ones bring the remedy. For example, Grossman and Krueger (1993, 1995) find inverted U-shape relationships for two measures of air pollution (sulphur dioxide and smoke) and several measures of water pollution (oxygen loss and concentrations of several heavy metals). Their evidence is confirmed in related work by Selden and Song (1994), Holtz-Eakin and Selden (1992), and the World Bank Development Report (1992) that finds similar patterns. The first goal of this paper is to present a simple static framework consistent with this evidence.

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Will growth in developed countries eventually cease because the environmental costs of increased production more than offset the benefits of higher consumption? Although cries from industry about the increasing burden of environmental regulations emphasize their role in raising costs and reducing productivity, the same regulations produce direct benefits for consumers. Whether society will choose a path with sustained growth depends on how the tradeoff between consumption and pollution evolves as the economy gets richer.[2] If the environmental costs of continued growth become sufficiently high, society will not be willing to pay them and growth will cease. But if increased productive capacity allows both consumption growth and improved environmental quality, then growth may continue without bound. A second goal of this paper is to determine whether environmental concerns will eventually limit growth. As we will see below, whether or not such a point is reached depends on the effects of pollution abatement on long-run rates of return.

Finally, can the path with an optimal balance between output growth and environmental protection be implemented in a decentralized economy? The third goal of this paper is to compare various regulatory schemes—direct controls, taxes, and vouchers—to see which can implement efficient paths. We will find that taxes and vouchers outperform direct regulation in terms of encouraging efficient capital accumulation.

Previous studies of growth and the environment have stressed intergenerational conflict of interest. John and Pecchenino (1994) study an overlapping generations model in which the young collectively tax themselves to make investments that improve the environment when they are old. They show that there may be multiple steady states, that equilibria are Pareto inefficient, and that there may be overinvestment in the environment. Jones and Manuelli (1994) study an overlapping generations model in which the young collectively choose a tax on pollution that will prevail when they are old. The tax has the (positive) direct effect of reducing pollution and the (negative) indirect effect of reducing the rate of return on capital. They show that for a certain class of preferences the tax rate and total pollution level are constant in the long run, and consumption grows at a constant rate.

Here it will be assumed that there is an infinitely lived representative household and that the political system acts in the interest of that household. Thus, intergenerational conflict of interest is ignored. The assumption of a long-lived dynasty, with perfect altruism between generations, seems very natural in this context. Evidence on savings and bequests (for example, see Laitner and Juster 1993) suggests a substantial degree of altruism between parents and children. In the present context we can expect this altruism to affect collective decisions about the environment as well as private savings decisions. Moreover, much of the public discussion of the environment and growth is cast in terms of the damaged planet we might leave to our children.

For the usual reasons heterogeneity in preferences and income is ignored, and the public sector decision is treated as a Ramsey problem. This approach is a convenient way of avoiding the issue of how the political system aggregates divergent views.

[2] See Jaffe et al., (1995) for an excellent discussion of the evidence on the costs of pollution abatement.

Income is evidently a primary factor determining attitudes toward environmental regulations, and the broad patterns of change over time will be mainly a response to rising incomes. The Ramsey approach is a simple, tractable way to capture this response. Admittedly, even in representative democracies there are many reasons why the government may not implement optimal policies. This is especially true for environmental regulations, since there are likely to be powerful special interests that are harmed by those regulations. An analysis of the political economy of environmental standards is an interesting area for further research, but for the long-run issues studied here the Ramsey solution provides a useful first step.

Some types of pollution dissipate very rapidly, so high pollution levels can be reduced quickly once a decision has been made to do so. For example, levels of sulfur dioxide and particulates in the air decline rapidly in the absence of new inflows, and the same is true for certain types of water pollution. In these cases it is reasonable to assume that the disutility of pollution is related to the flow of new pollutants. Other types of environmental degradation, such as deforestation and depletion of the ozone layer, are cumulative and very slowly self-correcting. In these cases it is more reasonable to assume that disutility is related to the cumulated stock of pollutants. Both approaches are considered here, and the conclusions are similar.

The analysis here has three parts. First a static model of environmental regulation is developed that is consistent with the available evidence relating per capita income and pollution levels. The technology allows a decision about how to use conventional factor inputs, with more productive use of those inputs generating more pollution. Equivalently, output is a function of conventional inputs and the quantity of pollution generated. Under these assumptions the dirtiest method of production is used if productive capacity is below a critical threshold and progressively cleaner methods as capacity rises above that threshold. Below the threshold total pollution increases linearly with income. Above the threshold its behavior depends on the elasticity of the marginal utility of consumption goods. For an elasticity greater than one the model matches the pattern found by Grossman and Krueger (1995): pollution is an asymmetric hump-shaped function of income, with an upper tail that declines relatively gradually.

For the second part of the analysis the same preferences and technology are embedded in three growth models: an Ak model in which growth is endogenous and two versions of a model with exogenous technological change. In one of these pollution is treated as a flow and in the other as a stock. In all three models the time path for total pollution displays the pattern suggested by the static model: if the elasticity of marginal utility exceeds one, total pollution first rises with income, then peaks and gradually declines. The models have quite different implications for long-run growth, however.

In the Ak model sustained growth is not optimal in the presence of pollution. The intuition for this result is very simple. As the capital stock grows society imposes ever stricter emissions standards, reducing the rate of return on capital. When the rate of return gets low enough there is no incentive for further accumulation.

With exogenous technical change there is a balanced growth path along which capital, output, and consumption grow at a common, constant rate, the emission standard becomes stricter at a constant rate, and the rate of return on capital remains constant, encouraging further accumulation. The tightening emission standard reduces the growth rate below what it would be in the absence of pollution, but growth is still positive. Moreover, the long-run behavior of the system is the same for the variants in which pollution is a flow and a stock.

All three growth models are studied as social planner's problems, in which the planner makes savings decisions as well as regulating pollution. For the third and final part of the analysis the issue of implementing these optima is studied. Specifically, the government is assumed to set environmental regulations and households to make savings decisions. The main conclusion is that tax and voucher schemes are capable of implementing the optima, but direct regulation is not. The intuition for this is as follows. The production technology can be written with capital and the total level of pollution as inputs. If pollution is regulated using a tax or a voucher system, then these two inputs are priced separately: the right to pollute has a market price that is entirely distinct from the rental rate for capital. Thus, the market return to capital measures the return on that input alone, and this price correctly guides savings decisions. By contrast, under direct regulation, ownership of a unit of capital confers the right to emit some pollution. Thus, the rental price of capital is the sum of the return to the capital itself and the implicit price of the pollution rights that accompany it. The bundled price, which exceeds the true return to capital, leads to excessive saving.

The rest of the paper is organized as follows. The static model is analyzed in Section 2, the Ak model and the two variants of the model with exogenous technological change are studied in Sections 3–5, implementation is treated in Section 6, and conclusions are discussed in Section 7. Mathematical derivations are contained in the Appendix.

2. A STATIC MODEL

In this section a static model of pollution is developed. The goal is to find restrictions on the technology and preferences consistent with the observed hump-shaped relationship between income and pollution levels described above.

Assume that consumption goods and pollution are joint products of a constant returns-to-scale technology. Let potential output y and actual output $c = yz$ both be measured on a per capita basis, where $z \in [0, 1]$ is an index of the technology used. Higher values for z yield more goods but also more pollution. Potential output is attained by using all productive resources in the dirtiest way, setting $z = 1$. Let $x = y\phi(z)$ be the total pollution generated when potential output is y and the production process z is used. Thus, z is an index of the emission rate for the production process. Assume that

$$\phi(0) = 0, \quad \phi'(0) = 0, \quad \phi(1) = 1, \quad \phi'(1) = \beta < \infty, \quad \phi''(0) > 0.$$

For fixed potential output, pollution is an increasing and convex function of actual output. No pollution is generated if no output is produced, pollution is finite even if the dirtiest technology is used, and the rate of increase in pollution as actual output rises toward its potential level is bounded above by β. Figure 1 displays production

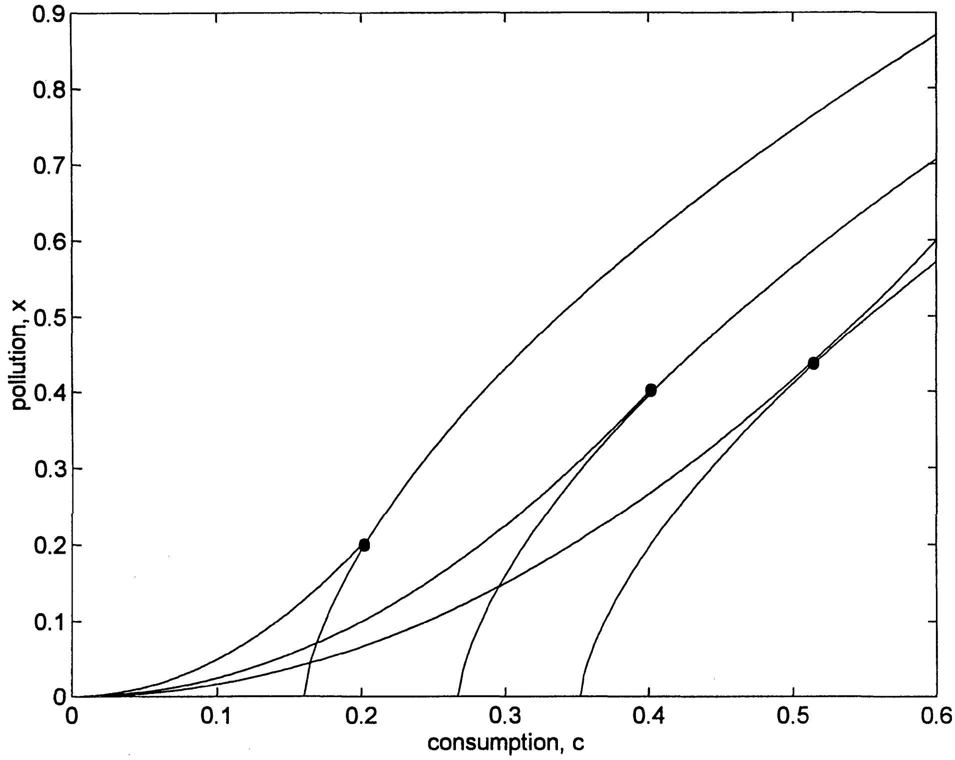

FIGURE 1

PREFERENCES AND TECHNOLOGY

possibility frontiers in $c-x$ space for three values of potential output y_i. Each is increasing and convex and has the form $(y_i z, y_i \phi(z))$, where y_i is fixed and z increases from zero to one. Potential output y_i increases toward the southeast.

Preferences over consumption c and pollution x are

$$U(c,x) = v(c) - h(x), \tag{1}$$

where v is strictly increasing and strictly concave, with $\lim_{c \to 0} v'(c) = +\infty$, and h is strictly increasing and strictly convex, with $\lim_{x \to 0} h'(x) = 0$. Figure 1 also displays three indifference curves, which are increasing and concave, with utility increasing toward the southeast.

Suppose that there is direct regulation of emissions, and competitive firms maximize profits within the constraints set by the law. Thus, the law sets a bound on the dirtiest technology that may be used, and all firms operate using that technology. (Other methods of implementation are studied in Section 6.) Given potential output y, the government sets the emission standard z to maximize the utility of the representative household. Hence the social planner's problem is

$$\max_{z \in [0,1]} v(yz) - h(y\phi(z)). \tag{2}$$

For fixed potential output y, the optimal technology $z^*(y)$ satisfies

$$v'(yz^*(y)) \geq h'(y\phi(z^*(y)))\phi'(z^*(y)), \quad \text{with equality if } z^*(y) < 1.$$

Figure 1 illustrates the solution for three levels of potential output. The small dots indicate the optima. For the two lower levels of potential output the optimum is at a corner, with $z^* = 1$ and $c = y = x$. For the highest level of potential output the optimum is interior, with $z^* < 1$ and $c, x < y$.

The general behavior of the optimal emission standard as potential income rises is very simple. As the first-order condition shows, at an interior optimum the opportunity cost of higher consumption in terms of pollution, ϕ', equals the marginal rate of substitution between consumption and pollution, v'/h'. Figure 2 displays the marginal benefits and marginal costs of pollution, $v'(y_i z)$ and $h'(y_i \phi(z))\phi'(z)$, (labelled MB_i and MC_i respectively), as functions of z, for two levels of potential

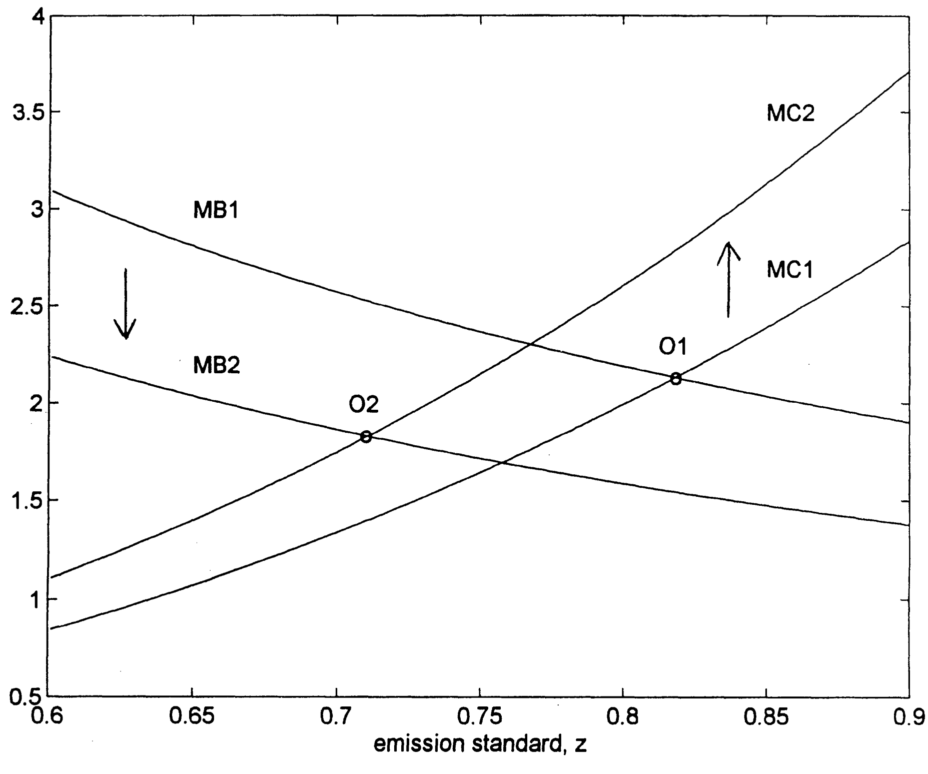

FIGURE 2

EFFECT OF GROWTH IN POTENTIAL OUTPUT

income, with $y_1 < y_2$. The marginal benefit curve is decreasing in z for fixed y and shifts downward as y increases. The marginal cost curve is increasing in z for fixed y and shifts upward as y increases. Define $\hat{y}$ by

$$\frac{v'(\hat{y})}{h'(\hat{y})} \equiv \phi'(1).$$

For potential income below $\hat{y}$ the marginal benefit and marginal cost curves do not intersect. The solution is at the corner $z = 1$, and the dirtiest technology is used. For potential income above $\hat{y}$ the solution is interior, and the emission standard becomes stricter as income rises, as shown in Figure 2. That is,

$$z^*(y) = 1, \quad y \le \hat{y},$$

$$z^{*\prime}(y) < 0, \quad y > \hat{y}.$$

In addition, consumption increases with potential income: $yz^*(y)$ is strictly increasing in y.[3]

We are also interested in the behavior of total pollution. Define

$$x^*(y) \equiv y\phi(z^*(y))$$

to be total pollution when the optimal technology is used. Clearly, total pollution increases with income below the critical income level $\hat{y}$. To see what happens above $\hat{y}$ it is useful to specialize to constant elasticity functions. Let

$$\phi(z) = z^{\beta}, \quad \beta > 1,$$

$$v(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}, \quad \sigma > 0,$$

$$h(x) = \frac{Bx^{\gamma}}{\gamma}, \quad \gamma > 1, B > 0.$$

Then the optimal emission standard $z^*(y)$ and total pollution level $x^*(y)$ are

$$z^*(y) = 1, \qquad \text{and } x^*(y) = y, \qquad \text{if } y \le \hat{y},$$

$$z^*(y) = (\hat{y}/y)^{\mu}, \quad \text{and } x^*(y) = \hat{y}^{\beta\mu} y^{1-\beta\mu}, \quad \text{if } y > \hat{y},$$

[3] The same conclusion holds for any utility function for which the marginal rate of substitution between consumption and pollution, $-U_1(c,x)/U_2(c,x)$, is strictly decreasing in c and x. A sufficient condition is that $U_{12} \le 0$.

where

$$\hat{y} = (B\beta)^{-1/(\sigma+\gamma-1)}, \quad \mu = \frac{\sigma+\gamma-1}{\sigma+\beta\gamma-1} \in (0,1).$$

Hence for $y > \hat{y}$ total pollution decreases with income if $\sigma > 1$, increases with income if $\sigma < 1$, and is constant for the logarithmic case $\sigma = 1$.

The intuition for this conclusion is as follows. Consumption can be written as $c = yf(x/y)$, where $f \equiv \phi^{-1}$. For the constant elasticity function $\phi(z) = z^{\beta}$, we have $c = y(x/y)^{1/\beta} = y^{1-1/\beta}x^{1/\beta}$. Thus, consumption goods are in effect produced with potential output y and total pollution x as inputs, and for these functional forms the technology is Cobb-Douglas. Stated in terms of x rather than z, the planner's problem is

$$\max_{x\in[0,y]} \frac{\left(y^{1-1/\beta}x^{1/\beta}\right)^{1-\sigma}-1}{1-\sigma} - \frac{Bx^{\gamma}}{\gamma},$$

so the condition for an optimum is

$$y^{(1-\sigma)(\beta-1)/\beta}x^{*}(y)^{(1-\sigma-\beta)/\beta} \geq \beta Bx^{*}(y)^{\gamma-1}, \quad \text{with equality if } x^{*}(y) < y.$$

The right side of this expression, the marginal cost of higher pollution, does not depend directly on y. The left side, the marginal benefit of higher output (consumption), is increasing in y if $\sigma < 1$, decreasing in y if $\sigma > 1$, and independent of y if $\sigma = 1$. The optimal level for total pollution behaves accordingly.

Thus, for $\sigma > 1$ the model is compatible with the two broad features of the data. At incomes below a critical level there are no pollution controls and total pollution increases with income. At incomes above the critical level emission standards become increasingly stringent and the total level of pollution declines.

Figure 3 displays $x^{*}(y)$, total pollution as a function of income. No attempt has been made to calibrate the model, but the qualitative pattern is illustrative. (The parameter values are $\sigma = 2$, $\beta = 2.0$, $\gamma = 1.5$, and $B = 1.0$.) The figure displays the inverted U-shape found by Grossman and Krueger (1995) and others. Notice, too, that while the relationship is single peaked, it is not well approximated by a quadratic function. A cubic approximation with a positive coefficient on the third term will do significantly better in fitting the upper tail. The regression results in Grossman and Krueger (1995) have exactly this form.

This one-good model can be interpreted as a reduced form of the framework in Copeland and Taylor (1994), which has multiple consumption goods and technologies that vary in terms of pollution intensity. Goods are indexed by $n \in [0,1]$. Each good is produced with a Cobb-Douglas technology, with capital and pollution as inputs. Let $\alpha(n)$ be the factor share for pollution in industry n, and assume that $\alpha(n)$ is continuous. In addition, suppose there is an upper bound λ on the input ratio for each good. Preferences over goods are Cobb-Douglas, with budget shares $b(n)$ that integrate to unity: $\int b(n)\,dn = 1$. Define the following indirect utility

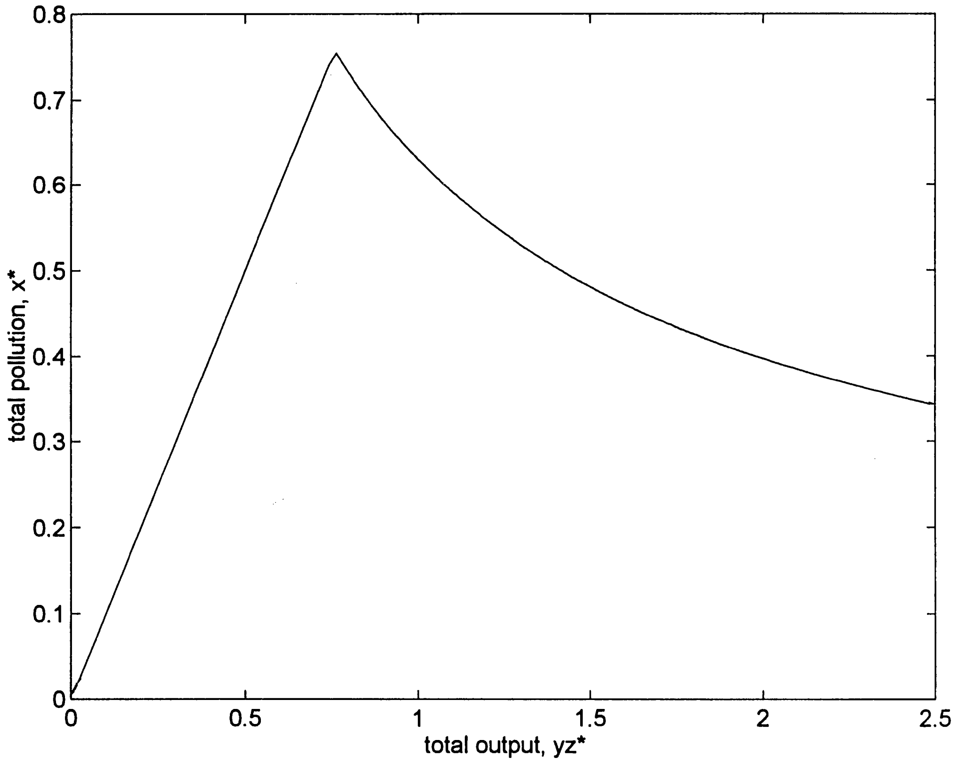

FIGURE 3

ACTUAL OUTPUT AND TOTAL POLLUTION

function, that has total capital y and total pollution x as arguments:

$$c(y,x) = \max \exp\left\{\int_0^1 b(n)\ln(C(n))\,dn\right\}$$

$$\text{s.t. } C(n) = Y(n)^{1-\alpha(n)}X(n)^{\alpha(n)}, \text{ all } n,$$

$$X(n) \le \lambda Y(n), \text{ all } n,$$

$$\int_0^1 Y(n)\,dn \le y,$$

$$\int_0^1 X(n)\,dn \le x.$$

Thus, $c(y,x)$ is the maximum utility from goods that can be attained with inputs (y,x). Clearly $c(y,x)$ is homogeneous of degree one, so $c(y,x) = yh(x/y)$, where h

is continuous, strictly increasing, and strictly concave. Define $z \equiv h(x/y)$, and note that $x = y\phi(z)$, where $\phi \equiv h^{-1}$ is continuous, strictly increasing, and strictly convex. Let $U(c, x)$ be the consumer's utility function over goods and pollution. Then given the resource endowment y, the social planner's problem is

$$\max_x U(c(y,x),x) = \max_x U(yh(x/y),x) = \max_z U(yz, y\phi(z)),$$

which is precisely the problem in (2) for the preferences in (1).

In the model with many goods, two types of adjustment take place as income rises. First, each individual commodity is produced with a cleaner technology, so in this sense the emission standard for each industry gets stricter. Second, the relative prices of more pollution-intensive goods rise, so the mix of goods changes, shifting towards those with less pollution-intensive technologies.

3. A DYNAMIC AK MODEL

In this section the preferences and pollution technology above are embedded into a one-sector endogenous growth model in which potential output is governed by a linear (Ak) technology. We are interested in the qualitative behavior of the emissions standard and total pollution level over time, and in whether the economy displays sustained growth.

As before, we consider the social planner's problem. In this case the planner chooses paths for consumption and for the technology in use to maximize the utility of the infinitely lived representative household. For convenience assume that capital does not depreciate. Then the planner's problem is to choose $\{c(t), z(t), t \geq 0\}$, to solve

$$\max \int_0^\infty e^{-\rho t}\left[\frac{c^{1-\sigma}-1}{1-\sigma} - \frac{B}{\gamma}(Akz^{\beta})^{\gamma}\right] dt$$

$$\text{s.t. } \dot{k} = Akz - c,$$

given $k(0) = k_0$. Assume that $A > \rho$, which is necessary and sufficient for sustained growth in the absence of environmental considerations.

The presence of a utility cost of pollution does not immediately rule out sustained growth. In fact, given any growth rate for capital, it is possible for the emission standard to improve at a rate that is fast enough so total pollution falls but slow enough so output rises. Specifically, total pollution is Akz^{β} and output is Akz, so the former declines and the latter grows if

$$0 < \frac{1}{\beta}\frac{\dot{k}}{k} < -\frac{\dot{z}}{z} < \frac{\dot{k}}{k}.$$

Nevertheless, although sustained growth is possible it is not optimal: the system converges to a steady state for any parameter values. Assume $A/\beta < A - \rho$, so the steady state displays binding environmental regulations. This case holds if households are patient (ρ is low), the output technology is productive (A is high), or the pollution technology is strongly convex (β is large).[4]

During the transition to the steady state, consumption and the emission standard satisfy

$$c = \lambda^{-1/\sigma}$$

$$z = \begin{cases} 1, & \text{if } \lambda \geq k^{\gamma-1}/m, \\ (\lambda m k^{1-\gamma})^{\psi}, & \text{if } \lambda < k^{\gamma-1}/m, \end{cases}$$

where λ is the shadow value of capital (the costate for k), and where m and ψ are functions of the technology and preference parameters. Over time the capital stock grows, consumption rises, and the costate λ falls. There is a critical level for the capital stock, defined by $m\lambda = k^{\gamma-1}$, where pollution controls come into play. Before the capital stock reaches this critical level the dirtiest technology is used ($z = 1$), and after it passes this level progressively cleaner technologies are used (z declines monotonically).

Total pollution during the transition is

$$x = Akz^{\beta} = \begin{cases} Ak, & \text{if } z = 1, \\ A\left((m\lambda)^{\beta} k^{\beta-1}\right)^{\psi}, & \text{if } z < 1. \end{cases}$$

Clearly it grows while $z = 1$. After the capital stock passes the critical level and z is declining, total pollution declines if and only if

$$\beta\frac{\dot{\lambda}}{\lambda} + (\beta - 1)\frac{\dot{k}}{k} < 0.$$

This condition holds in the neighborhood of the steady state if and only if $\sigma > (\beta - 1)/\beta$. Hence $\sigma > 1$ is sufficient to ensure that total pollution displays an inverted U-shape pattern over time, growing in the early stages of development and declining as the economy approaches the steady state.

The intuition for why growth ceases is related to the rate of return on capital. In the absence of pollution the real rate of return on capital is constant: $r = A$. Since by assumption $A > \rho$, the interest rate is high enough to make additional investment always attractive. In the current model, output is produced with the capital stock and the aggregate pollution level as inputs, $y = (Ak)^{1-1/\beta}x^{1/\beta}$. Hence the marginal product of capital is

$$r = \left(\frac{\beta - 1}{\beta}\right)A^{1-1/\beta}k^{-1/\beta}x^{1/\beta}.$$

[4] If $A - \rho \leq A/\beta$, then growth ceases before pollution regulations come into play. This case is not interesting for our purposes.

Thus, the real return to capital is not the constant A, but something that declines with capital for any fixed level of pollution. The only way to maintain the real rate of return as the capital stock grows is by letting total pollution grow in proportion to capital. But utility is concave in consumption and convex in total pollution, so the path where consumption and total pollution grow at the same rate is easily dominated. In particular, since total pollution is $x = Akz^{\beta}$, the real interest rate can be written as

$$r = \left(\frac{\beta - 1}{\beta} \right) Az.$$

As the capital stock grows the optimal emission standard becomes stricter, reducing the real rate of return. When the emission standard gets strict enough, capital accumulation ceases. In the steady state the rate of return and emission standard are

$$r_{ss} = \left(\frac{\beta - 1}{\beta} \right) Az_{ss} = \rho.$$

Figure 4 displays time paths for this model. As before, only the qualitative patterns are interesting. (The parameter values are $\rho = 0.04$, $\sigma = 2.0$, $B = 1.0$, $\gamma = 1.2$, $A = 0.12$, and $\beta = 2.0$.) The first panel displays the emission standard, z. There is a critical date, call it T, before which there is no regulation of pollution, $z = 1$. After date T the emission standard tightens quickly for a while and then declines gradually towards the steady state. The second panel shows total pollution relative to its steady-state value, x/x_{ss}. It rises sharply towards its peak at date T, and then declines slowly.

The third panel shows the real interest rate and consumption growth. The interest rate is constant at $r = A(\beta - 1)/\beta$ before date T, and declines towards its steady-state level $r_{ss} = \rho$ thereafter. Consumption growth roughly tracks the interest rate, $g_c \approx (r - \rho)/\sigma$. The last panel shows capital and output growth. Since $\dot{y}/y = \dot{k}/k + \dot{z}/z$, output and capital grow at a common rate before date T. After date T output growth declines sharply as z falls. Capital growth has a brief upturn just before date T that is rather puzzling.

4. EXOGENOUS TECHNICAL CHANGE

In this section the same preferences and pollution technology are embedded into a one-sector growth model with exogenous technical change. The presence of technical change means there cannot be a steady state; instead there is a balanced growth path along which capital, output, and consumption grow at a common, constant rate. As before we are interested in how the emission standard and total pollution level evolve over time and how the presence of pollution affects the long-run growth rate.

Assume that the production technology is Cobb-Douglas, with share $0 < \alpha < 1$ for capital, that technical change occurs at the constant rate $g > 0$, and that capital

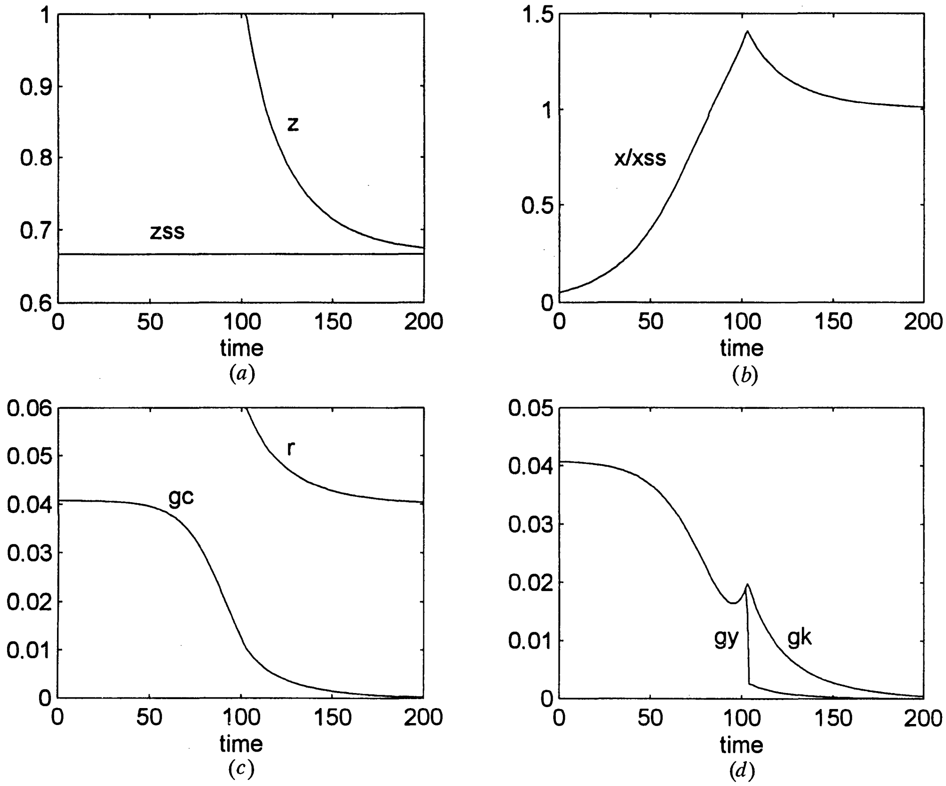

FIGURE 4

(a) EMMISION STANDARD. (b) TOTAL POLLUTION. (c) INTEREST RATE AND CONSUMPTION GROWTH. (d) CAPITAL AND INCOME GROWTH.

depreciates at the constant rate $\delta \geq 0$. Then the social planner's problem is

$$\max \int_0^\infty e^{-\rho t}\left[\frac{c^{1-\sigma}-1}{1-\sigma} - \frac{B}{\gamma}\left(Ae^{gt}k^\alpha z^\beta\right)^\gamma\right] dt$$

$$\text{s.t. } \dot{k} = Ae^{gt}k^\alpha z - \delta k - c,$$

given $k(0) = k_0$.

Along the optimal path consumption and the emission standard satisfy

$$c = \lambda^{-1/\sigma},$$

$$z = \begin{cases} 1, & \text{if } \lambda \geq \left(e^{gt}k^\alpha\right)^{\gamma-1}/m, \\ \left[\lambda m\left(e^{gt}k^\alpha\right)^{1-\gamma}\right]^\psi, & \text{if } \lambda < \left(e^{gt}k^\alpha\right)^{\gamma-1}/m, \end{cases}$$

where as before m and ψ are functions of the parameters and λ is the shadow value (costate) for capital. Capital grows, consumption rises, and λ falls along the optimal path. As in the Ak model there is a critical point, here defined by the condition $m\lambda = (e^{gt}k^{\alpha})^{\gamma-1}$, before which the dirtiest technology is used ($z = 1$), and after which the emission standard becomes increasingly strict (z declines monotonically). Note that here the critical point involves the technology level as well as the size of the capital stock.

The (common) growth rate of capital, output, and consumption along the balanced path is

$$g_k = \frac{\gamma(\beta - 1)g}{(1-\alpha)\gamma(\beta - 1) + (\sigma + \gamma - 1)}, \tag{3}$$

so

$$0 < g_k < \frac{g}{1-\alpha}.$$

The transversality condition holds if $(1-\sigma)g_k < \rho$, which is satisfied if $\sigma \geq 1$ or if g_k is not too large. In this case the balanced path is locally stable. If we view the technical change as labor augmenting, then effective labor grows at the rate $g/(1-\alpha)$. In the absence of environmental considerations, output, consumption, and the capital stock would also grow at this rate. In the presence of pollution, they grow more slowly.

Writing z in terms of capital and output, we obtain

$$z = y\left(Ae^{gt}k^{\alpha}\right)^{-1}.$$

Along the balanced path the emission standard $z < 1$ improves at a constant rate that offsets the difference between the rate of labor-augmenting technical change, $g/(1-\alpha)$, and the growth rate of capital, g_k, so output grows at the same rate as capital:

$$g_z \equiv \frac{\dot{z}}{z} = (1-\alpha)g_k - g < 0. \tag{4}$$

Total pollution is

$$x = Ae^{gt}k^{\alpha}z^{\beta} = \begin{cases} Ae^{gt}k^{\alpha}, & \text{if } z = 1, \\ \left(Ae^{gt}k^{\alpha}\right)^{1-\beta}y^{\beta}, & \text{if } z < 1. \end{cases}$$

Clearly it increases prior to the critical date when an emission standard is first imposed. Along the balanced path it grows at the rate

$$g_x = \beta g_k - (\beta - 1)(g + \alpha g_k) = \frac{1-\sigma}{\gamma}g_k, \tag{5}$$

so $g_x < g_k$, and pollution grows more slowly than output. Total pollution declines along the balanced path if (and only if) $\sigma > 1$.

The behavior of the interest rate again provides the intuition for the long-run growth rate. With technical change, sustained growth in the capital stock and continued tightening of the emission standard are compatible with a constant interest rate. As before, output can be written as

$$y = \left(Ae^{gt}k^{\alpha}\right)^{1-1/\beta} x^{1/\beta},$$

so the marginal product of capital is

$$r = \frac{\alpha(\beta - 1)}{\beta}\frac{y}{k}.$$

Along the balanced path the ratio y/k is constant, so the interest rate is also constant.

Figure 5 displays time paths for this model. (The parameter values are $\rho = 0.04$, $\sigma = 2.0$, $B = 1.0$, $\gamma = 1.2$, $A = 1.6$, $\alpha = 0.40$, $\delta = 0.06$, $\beta = 3.0$, and $g = 0.024$.) Of the displayed period, about half is a transition phase and about half represents growth along the balanced path. The first panel displays the emission standard. As before there is a critical date T before which the dirtiest technology is used, $z = 1$, and after which the emission standard becomes progressively stricter. The second panel shows total pollution, which peaks at date T and declines thereafter. The third panel shows the interest rate and consumption growth. The interest rate declines during the transition period and is constant along the balanced path, and consumption growth roughly tracks the interest rate. The fourth panel shows capital and output growth. Both decline during the transition, falling towards their long run value of 2%. As before, output growth declines sharply just after date T. Capital and output growth again display a curious upturn just before date T.

5. POLLUTION STOCKS

For some types of environmental degradation, like deforestation and depletion of the ozone layer, it is the cumulative effect of past actions that affects utility at each date. To capture this idea assume that pollution accumulates as a stock that affects utility. Assume that the stock decays at a fixed rate, so environmental damage is (possibly) self-correcting, although the rate at which that occurs may be slow. As in the previous section, assume that growth is driven by exogenous technical change.

In this case the social planner's problem is

$$\max \int_0^{\infty} e^{-\rho t}\left[\frac{c^{1-\sigma} - 1}{1 - \sigma} - \frac{B}{\gamma}X^{\gamma}\right] dt$$

$$\text{s.t. } \dot{k} = Ae^{gt}k^{\alpha}z - \delta k - c,$$

$$\dot{X} = Ae^{gt}k^{\alpha}z^{\beta} - \eta X,$$

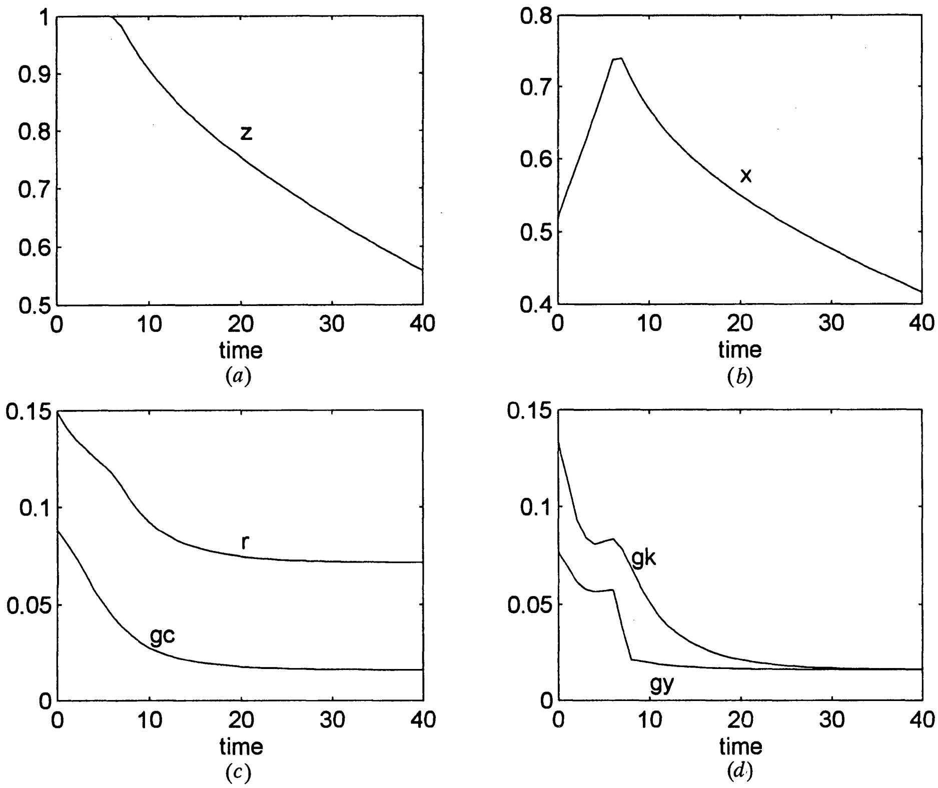

FIGURE 5

(*a*) EMMISION STANDARD. (*b*) TOTAL POLLUTION. (*c*) INTEREST RATE AND CONSUMPTION GROWTH. (*d*) CAPITAL AND INCOME GROWTH.

given $k(0) = k_0$ and $X(0) = X_0$, where $X(t)$ is the stock of pollution at date t, and $\eta \geq 0$ is the rate at which that stock decays.

The dynamics are more complicated in this case than in the previous one, since they depend on the initial stock of pollution as well as the initial capital stock. For plausible initial conditions, however, the transition to the balanced path is similar to the one in the previous section. The conditions for an optimum in this case include

$$c = \lambda^{-1/\sigma},$$

$$z = \begin{cases} 1, & \text{if } \lambda \geq \beta\mu, \\ (\lambda/\mu\beta)^{1/(\beta-1)}, & \text{if } \lambda < \beta\mu, \end{cases}$$

where λ and μ are the costates for k and X. Thus, the dirtiest technology is used if the shadow price of capital, λ, is sufficiently high relative to the shadow price of the pollution stock, μ.

Suppose the initial stocks of both pollution and capital are small. Then along the transition path μ rises and λ falls, so the ratio λ/μ declines monotonically. In this case the behavior of the emission standard is the same as in the earlier models: there is a critical date, here defined by $\lambda/\mu\beta = 1$, before which the dirtiest technology is used ($z = 1$) and after which the standard becomes progressively stricter (z declines monotonically). Both the gross inflow of new pollution and the stock of pollution rise before the critical date.

The growth rates along the balanced path for this model are exactly the same as those in the previous section. Capital, consumption, and output grow at rate g_k and the emission standard improves the rate g_z, given by (3) and (4). Moreover, the growth rate for total pollution, now a stock, is also the same as before,

$$g_X = \frac{1-\sigma}{\gamma} g_k. \tag{6}$$

Hence the stock declines along the balanced path if and only if $\sigma > 1$.

To see why the growth rate of pollution is unchanged, and why it is independent of the decay rate η, let $x(t)$ denote the gross inflow of new pollution at date t, and write the law of motion for the stock of pollution as

$$\frac{\dot{X}(t)}{X(t)} = \frac{x(t)}{X(t)} - \eta.$$

If the stock grows at the constant rate g_X, then the ratio $x(t)/X(t) = g_X + \eta$ must be constant, so the inflow of new pollution grows at the same rate as the stock. Thus, along the balanced path $g_x = g_X$. Notice that the decay rate η does affect the steady state ratio x/X, just as the depreciation rate for capital typically affects the capital–output ratio but not the long-run growth rate of capital. Also notice that the model in the previous section can be viewed as the limiting case as $\eta \to \infty$.

A solution of this type exists if and only if

$$-\gamma\eta < (1-\sigma)g_k < \rho. \tag{7}$$

The second inequality, which is required for the transversality conditions, is the same as before, and for $\sigma > \alpha$ the roots are real and the balanced path is locally stable. For $\sigma > 1$, the case of interest, the first inequality holds if the decay rate η is not too small. Thus, pollution must be self-correcting. The decay rate does not have to be very large, however. For example, if the growth rate of capital is 1.5% and the preference parameters are $\sigma = 3$ and $\gamma = 2$, then (7) holds if $\eta > 1.5\%$.

6. IMPLEMENTATION

Up to this point the problem of environmental regulation has been treated as a social planner's problem, with the planner choosing optimal paths for consumption and capital accumulation as well as the emission standard. In this section the problem of implementing those optima is studied. For simplicity, only the static and

Ak models are considered. The issues involved for the latter apply to the other two growth models as well.

Assume that the government has an instrument to regulate pollution and that consumption and saving are determined by the interaction of households and firms in competitive markets. Three government instruments are considered: direct regulation of the emission standard, a voucher system, and a pollution tax.

There is a continuum of identical households, with total mass one. Each owns capital, which it rents to firms. The household also receives wages, the profits generated by firms, and lump sum rebates distributed by the government, if there are any. The household makes consumption and savings decisions.

There is also a continuum of firms. Since the technology displays constant returns to scale, the precise number of firms is irrelevant. For simplicity, take their total mass to be one. Firms maximize profits, taking input prices and tax rates, if applicable, as given. They make decisions about how much capital to employ, what technology (emission rate) to use, and, consequently, how much pollution to generate, within the bounds set by the law.

First consider the static model. To keep the notation consistent with what comes later in this section, let $y = Ak$, and let $z^*(k)$ and $x^*(k) = Akz^*(k)^{\beta}$ be the optimal emission standard and total pollution level, as defined in Section 2. Note that $x^*(k) < Ak$ if and only if $z^*(k) < 1$. In the static case the optimum can be implemented by any of the three instruments.

Suppose the government sets an emission standard directly, by regulating the dirtiest technology that may be used. Then given society's capital stock k, it should choose the bound $z^*(k)$, and all firms will operate using that technology. The only market is a rental market for capital, and the rental rate is the marginal product of capital, so $r = Az^*(k)$. Firms earn no profits. Households consume their income, which consists of the returns to capital, so $c = rk = Akz^*(k)$.

Alternatively, suppose the government distributes pollution vouchers uniformly across firms and then allows a secondary market in those vouchers. Given the capital stock k, the quantity of vouchers it should distribute is $x^*(k)$. In this economy there are two input markets, for capital and vouchers. Let r be the rental rate for capital and τ the price of a voucher. Each firm chooses its input levels (K, X) to maximize profits, so its problem is

$$\max_{X \le AK} \, (AK)^{1-1/\beta} X^{1/\beta} - rK - \tau[X - x^*(k)]. \tag{8}$$

The competitive equilibrium prices follow immediately from the first-order and break-even conditions, evaluated at the market clearing quantities $K = k$ and $X = x^*(k)$:

$$\tau \le \frac{1}{\beta}\left(\frac{x^*(k)}{Ak}\right)^{1/\beta - 1},$$

$$r \ge \frac{\beta - 1}{\beta} A \left(\frac{x^*(k)}{Ak}\right)^{1/\beta}, \quad \text{with equality if } x^*(k) < Ak,$$

and

$$rk + \tau x^*(k) = (Ak)^{1-1/\beta} x^*(k)^{1/\beta}.$$

If pollution controls are binding, in the sense that $x^*(k) < Ak$, then the first-order conditions uniquely determine prices:

(9a)
$$\tau = \frac{1}{\beta} z^*(k)^{1-\beta},$$

and

(9b)
$$r = \frac{\beta - 1}{\beta} Az^*(k), \quad \text{if } x^*(k) < Ak.$$

Otherwise there is some indeterminacy and the break-even condition must be used. In this case

(10a)
$$\tau \in \left[0, \frac{1}{\beta}\right],$$

and

(10b)
$$r = A(1 - \tau), \quad \text{if } x^*(k) = Ak.$$

Notice that in the latter case the interest rate can lie anywhere in the interval $r \in [A(1 - 1/\beta), A]$. At the prices given by (9) or (10), each firm chooses the pair $(k, x^*(k))$, and its profits are equal to the market value of its vouchers:

$$\pi = \tau x^*(k) = \begin{cases} Akz^*(k)/\beta, & \text{if } x^*(k) < Ak, \\ (A - r)k, & \text{if } x^*(k) = Ak. \end{cases}$$

Households consume their income, which consists of profits plus returns to capital, so

$$c = \pi + rk = \begin{cases} Akz^*(k), & \text{if } x^*(k) < Ak, \\ Ak, & \text{if } x^*(k) = Ak. \end{cases}$$

Hence the optimum can be implemented with a voucher plan as well. If vouchers are distributed to households rather than firms, then π is the household's income from the sale of its vouchers and the equilibrium is otherwise the same.

A pollution tax can also implement the optimum, and it does so with essentially the same prices as a voucher scheme. Suppose the capital stock is k and the government taxes pollution at the rate τ. Then the representative firm's problem is again (8), except that the term $\tau x^*(k)$, which does not affect the maximization, disappears. Hence the first-order and break-even conditions are the same as above. Thus, if $x^*(k) < Ak$, the tax and interest rate are those in (9), while if $x^*(k) = Ak$ the government can choose any of the tax rates in (10a) and the associated interest rate satisfies (10b). Firms choose the input pair $(k, x^*(k))$, and the revenue from the tax, which is rebated to households, is $\tau x^*(k) = \pi$. Hence households have the same income and consumption as under the voucher scheme.

Two features of these implementation schemes are noteworthy. First, the rate of return on capital is higher under direct regulation than it is under the voucher or tax schemes. Under direct regulation pollution vouchers are in effect tied to capital: each unit of capital entitles the owner to emit $Az^*(k)^\beta$ units of pollution. Thus, the rental price of capital is the sum of the values of the capital itself and the pollution rights that come with it. In the static context this bundling has no consequences, since the quantity of capital is fixed. In the context of a growth model, however, the interest rate affects savings decisions, so this bundling is likely to have real effects.

Second, if pollution controls are nonbinding, then under the voucher and tax schemes the rate of return on capital is indeterminate within some range. In a static context this indeterminacy has no real effects. In a growth model, however, implementing the optimal savings decisions will require the 'right' path for the interest rate.

With these two caveats in mind, consider the problem of implementing the optimal path for the Ak growth model. Let k_0 be given, let $\{c^*(t), z^*(t), k^*(t), \lambda^*(t), t \geq 0\}$ be the solution to the social planner's problem, and let $x^*(t) \equiv Ak^*(t)z^*(t)^\beta$ be the path for total pollution.

Suppose the government wants to implement the optimum through a pollution tax. Households make savings decisions, taking as given the rate of return on capital, the lump sum subsidy, and the total pollution level. Their problem is

$$\max \int_0^\infty e^{-\rho t}\left[\frac{c(t)^{1-\sigma}-1}{1-\sigma} - \frac{B}{\gamma}x(t)^\gamma\right]dt$$

$$\text{s.t. } \dot{k}(t) = r(t)k(t) + \pi(t) - c(t),$$

given $\{r(t), \pi(t), x(t), t \geq 0\}$, and $k(0) = k_0 > 0$, where π denotes the subsidy. The conditions for a maximum are

$$c(t)^{-\sigma} = \Lambda(t),$$

$$\frac{\dot{\Lambda}(t)}{\Lambda(t)} = \rho - r(t),$$

where Λ is the shadow price (costate) for capital. Comparing these conditions with those for the social planner's problem (in the Appendix), we see that the optimum is implemented if and only if the paths for the costates, and hence for consumption, are identical. Moreover, the costates match if and only if the interest rate satisfies

$$\text{(11)} \qquad r^*(t) = \begin{cases} A\left[1 - k^*(t)^{\gamma-1}/\beta m \lambda^*(t)\right], & \text{if } z^*(t) = 1, \\ Az^*(t)(1 - 1/\beta), & \text{if } z^*(t) < 1. \end{cases}$$

The firm's problem at each date t is a static one: it maximizes profits, taking as given the rental rate on capital, $r(t)$, and the pollution tax, $\tau(t)$. Its problem is exactly the same as before, so to implement the optimum the tax rate and interest rate must satisfy (9) or (10). For dates when $z^*(t) < 1$, the interest rate in (11) agrees with the one in (9b), and (9a) supplies the tax rate. For dates when $z^*(t) = 1$, (10a) and (10b) together require that $r(t) \in [A(1 - 1/\beta, A]$. The interest rate in (11) lies in this interval, and (10b) supplies the tax rate. Hence

$$\tau^*(t) = \begin{cases} k^*(t)^{\gamma-1}/\beta m \lambda^*(t), & \text{if } z^*(t) = 1, \\ z^*(t)^{1-\beta}/\beta, & \text{if } z^*(t) < 1. \end{cases} \tag{12}$$

Tax revenue, which is rebated to households as a lump sum subsidy, is

$$\pi^*(t) = \tau^*(t) x^*(t), \text{ all } t.$$

At the prices $\{r^*, \tau^*\}$ in (11) and (12), the input pair $\{k^*, x^*\}$ is optimal for the firm at each date t, and the resulting paths for the emission standard and tax revenue are z^* and π^*. Given $\{r^*, \pi^*, x^*\}$, the values $\{c^*, k^*, \lambda^*\}$ solve the household's problem. Hence the social optimum is a competitive equilibrium for the tax policy τ^*.

Moreover, the optimal allocation is the unique equilibrium for this policy. To see this note that for the tax policy in (12), profit maximization implies that the interest rate satisfies (11). At these prices, $k^*(t)/x^*(t)$ is the unique profit-maximizing input ratio at each date t. Equivalently, $z^*(t)$ is the unique technology choice. In addition, since intertemporal preferences are homothetic, given the path r^* for the interest rate, the household's optimal consumption stream satisfies $\{\theta c^*(t),\ t \geq 0\}$, where $\theta > 0$ depends on the present value of the subsidy stream. Suppose that the present value of the actual subsidy stream $\{\tau^* x\}$ were to exceed the present value of the optimal stream $\{\tau^* x^*\}$. Then $\theta > 1$, and the resource constraint would imply

$$\dot{k}(t) = Ak(t) z^*(t) - \theta c^*(t).$$

Hence $k(t) < k^*(t)$, all t, so $x(t) < x^*(t)$, all t, contradicting the assumption that the present value of $\{\tau^* x\}$ exceeded the present value of $\{\tau^* x^*\}$. A similar argument shows that $\theta < 1$ is not possible. Hence the social optimum is the unique competitive equilibrium for the tax policy in (12). It is also clear that no other tax policy implements the optimum: the social optimum requires (11), which can obtain only if (12) holds.

Next consider a voucher scheme. Suppose the government distributes $x^*(t)$ pollution vouchers at each date t. The household's and firm's problems are the same as before, so it is straightforward to show that the quantities $\{c^*, k^*\}$, together with the prices $\{r^*, \tau^*\}$ in (11) and (12), and profit stream $\{\pi^*\}$, constitute an equilibrium.[5]

[5] Uniqueness is a little more complicated in this case. Clearly there are no other prices that support $\{c^*, k^*, x^*\}$ as equilibrium quantities. It is not so clear whether there are other paths for consumption, capital, and prices that, together with x^*, constitute a competitive equilibrium.

Moreover, it is clear that $\{x^*\}$ is the only voucher policy for which the optimum is a competitive equilibrium.

Finally, consider direct regulation. Suppose the government sets the emission standard $z^*(t)$ at each date t. Then the break-even condition for the firm implies that the return on capital at date t is $r(t) = Az^*(t)$. Since this is clearly different from the path for the interest rate in (11), the household will *not* choose $\{c^*, k^*\}$. Hence direct regulation cannot attain the optimum for the dynamic economy.

Two lessons emerge. First, dynamic considerations provide an additional reason, quite distinct from those that have been emphasized in static models, to prefer pollution taxes or pollution vouchers over direct regulation of emissions. Under tax or voucher schemes there are separate markets for capital and for the right to pollute. The rate of return on capital determines the savings behavior of households, and the ratio of the two prices determines the input mix chosen by firms. Under direct regulation the two markets are tied together. If an emission standard of $\hat{z}$ is imposed directly, then all capital is operated using that standard, and each additional unit of capital entitles the owner to emit $A\hat{z}^{\beta}$ additional units of pollution. As in the static model, the market rental rate for capital under direct regulation 'bundles' the true return to capital and the value of the additional pollution rights that accompany it. This bundling does not show up in the behavior of firms, since their input mix is regulated directly. It does show up in the behavior of households, however; the market return on capital is the wrong one for guiding savings behavior.

Second, it is interesting, and perhaps surprising, that under the tax or voucher policies that implement the optimum, the tax rate or voucher price τ^* is strictly positive even during the period when firms use the dirtiest technology. That is, implementing the optimum requires that firms pay a positive price for emissions, even during the period when the charge has no effect on the emission rate z^* they choose to use or on their total emission of pollutants x^*. The positive price on pollution during this period is needed to reduce the rate of return on capital, thereby affecting the incentives for new investment. Thus, a crucial feature of this price is that to be effective it must be anticipated in advance by potential investors.

7. CONCLUSIONS

The optimal regulation problems analyzed here can be interpreted, roughly, as positive models of democratic societies in which income is not too unequally distributed. In such societies the government's objective is to maximize the utility of the representative household, as here. Moreover, implementing these optima does not require extraordinary power on the part of the government: they can be implemented with standard instruments like a pollution tax or a voucher system. Thus, the problems studied here are Ramsey problems in which the instruments available to the government are sufficient to attain the first-best.

Much of the economic intuition for the observed hump-shaped relationship between income and pollution levels comes from the static model. The cost of higher pollution is a direct utility cost, so for additively separable preferences it is independent of income. The benefit, however, is the higher output (consumption) it allows, which depends on the level of conventional inputs. For low levels of those inputs, output is low and the marginal utility of consumption is high. Hence the benefit from additional pollution more than offsets the cost, at the margin, even when pollution is at the maximum. Thus, at low income levels the dirtiest and most productive technology is used, and pollution increases with income.

At higher levels of conventional inputs there are two possibilities. If the marginal utility of consumption is elastic, then pollution and conventional inputs are substitutes in the sense that an increase in the latter reduces the marginal value of the former. In this case pollution declines with income at high-income levels. The observed hump-shaped relation between income and pollution is consistent with such preferences. If the marginal utility of consumption is inelastic, pollution and conventional inputs are complements and pollution increases with income over the entire range.

Note that an implication of this model is that the hump shape is a consequence of the elasticity parameter for consumption goods, not for pollution. Thus, if additive separability is approximately correct, all types of pollution should display the hump shape, although the peaks may occur at different income levels.

For elastic preferences, all the growth models studied here display the inverted U-shape relationship as income grows over time. In this sense all the models are consistent with the time series and cross sectional data that display that pattern. The long-run growth implications of the models differ, however.

In the Ak model environmental effects imply that growth will eventually cease, since the stricter emission standards that accompany growth reduce the rate of return on capital, eventually pushing it below the level required to encourage further accumulation. In the models with exogenous technical change, sustained growth in capital and continued tightening of the emission standard are compatible with a constant rate of return. Hence the economy displays balanced growth in the long run, albeit at a slower pace than it would enjoy in the absence of environmental effects.

To conclude that the prospects for sustained growth depend on whether growth is exogenous or endogenous would be misleading, however. The key issue is whether environmental regulations reduce the rate of return below the rate of time preference. It should not be difficult to construct endogenous growth models with both human and physical capital, like those in Lucas (1988), that can display either type of behavior. Specifically, with constant returns to scale the model would behave like a two-factor version of the Ak model, and with a sufficiently strong positive external effect from human capital like the model with exogenous technological change.

Regulatory mechanisms that establish a market price for pollution, as taxes and vouchers systems do, have long been advocated over direct regulation in static settings. With heterogeneous firms, price-based mechanism are preferred because they require less information on the part of the government. To attain the first best under direct regulation, the government must know the cost function for each firm. To attain the optimum under a tax or voucher system, it only needs to know the aggregate cost function; it can then choose the appropriate price or quantity and let the market allocate total emissions efficiently across firms.[6]

[6] See Baumol and Oates (1979, Ch. 16–17, and 1988, Ch. 11–12).

In a dynamic setting, tax and voucher schemes have an additional advantage: under these systems the market interest rate produces the correct incentives for capital accumulation. With a pollution tax or a voucher system, emissions have a market price that is entirely separate from the return to capital. Hence the market return to capital is an accurate measure of the incremental value of investment and provides the correct incentive to save. With direct regulation, ownership of capital brings with it the right to emit a specified level of pollution. Thus, the market return to capital is the sum of its real economic return and the value of the associated pollution rights. The bundled price overstates the return to investment and provides the wrong incentives for accumulation.

It is interesting that the optimal tax and voucher schemes require a positive price for pollution during the phase when the dirtiest technology is used. The *only* role of this positive price is to reduce the return to capital, thereby reducing the incentives to save. Thus, it is crucial not only that the tax be imposed, but that it be anticipated in advance, so that savings decisions will be guided accordingly.

The results here suggest several areas for further research. First, it would be interesting to analyze the behavior of a model with endogenous technical change. In particular, it would be nice to have an endogenous technology model in which technical progress could be aimed at developing cleaner technologies. A key issue is whether the market provides sufficient incentives to develop pollution abatement technologies.

It would also be interesting to explore the political economy issues raised by environmental regulation. As noted above, there are likely to be powerful special interest groups—firms in polluting industries—that have much to gain by avoiding regulation. If these groups influence government policy, which surely they do, then the Ramsey solution must be modified.

The analysis here was of a single economy, which might be interpreted as a country or region. Many types of air and water pollution spill over across national or regional boundaries, however. This raises the question of how pollution can effectively be regulated in the presence of such spillovers: are tax or voucher schemes useful, are direct transfer payments required, do the conclusions change if capital can move across regions, and so on.

Another important issue is population density. In the analysis above, population size was taken as exogenous and constant. But population growth is clearly related to per capita income, albeit in ways that are not yet well understood. In addition, economic development typically entails a large shift of population from rural to urban areas, where pollution is more concentrated. Thus, income has indirect effects, as well as the direct one stressed here. On the empirical side, the rural–urban population shift should be taken into account in measuring increased exposure to pollutants as income rises.

Most of the world's population resides in countries with per capita incomes below the level where pollution peaks and emission controls come into play. Many of these countries are developing rapidly, and their emission of pollutants is growing apace. Questions about how quickly and by what means these emissions should be regulated will only become more important as their growth continues.

APPENDIX

Section 1. The conditions for a maximum for the Ak model are

$$c = \lambda^{-1/\sigma},$$

$$z = \begin{cases} 1, & \text{if } \lambda \geq k^{\gamma-1}/m, \\ (\lambda m k^{1-\gamma})^{\psi}, & \text{if } \lambda < k^{\gamma-1}/m, \end{cases}$$

$$\frac{\dot{k}}{k} = \begin{cases} A - k^{-1}\lambda^{-1/\sigma}, & \text{if } \lambda \geq k^{\gamma-1}/m, \\ A(\lambda m k^{1-\gamma})^{\psi} - k^{-1}\lambda^{-1/\sigma}, & \text{if } \lambda < k^{\gamma-1}/m, \end{cases}$$

and

$$\frac{\dot{\lambda}}{\lambda} = \begin{cases} \rho - A + (A/\beta)(\lambda m k^{1-\gamma})^{-1}, & \text{if } \lambda \geq k^{\gamma-1}/m, \\ \rho - (\beta - 1)(A/\beta)(\lambda m k^{1-\gamma})^{\psi}, & \text{if } \lambda < k^{\gamma-1}/m, \end{cases}$$

where

$$m = \frac{1}{\beta B A^{\gamma-1}}, \quad \text{and} \quad \psi = \frac{1}{\beta\gamma - 1} > 0.$$

If $A/\beta < A - \rho$, the steady state values are

$$z_{ss} = \frac{\beta\rho}{(\beta - 1)A} < 1, \quad k_{ss} = \left[m A^{-\sigma} z_{ss}^{1-\beta\gamma-\sigma} \right]^{\phi}, \quad \text{and} \quad c_{ss} = A k_{ss} z_{ss},$$

where

$$\phi = \frac{1}{\sigma + \gamma - 1} > 0.$$

To check stability and to characterize the behavior of total pollution near the steady state, define

$$p = \ln\left(\frac{k}{k_{ss}} \right) \quad \text{and} \quad q = \ln\left(\frac{\lambda}{\lambda_{ss}} \right),$$

so the log-linear approximation to the law of motion is

$$\begin{pmatrix} \dot{p} \\ \dot{q} \end{pmatrix} \approx \begin{pmatrix} [(1-\gamma)\psi + 1] k_{ss}^{-1} \lambda_{ss}^{-1/\sigma} & (\psi + 1/\sigma) k_{ss}^{-1} \lambda_{ss}^{-1/\sigma} \\ \rho\psi(\gamma - 1) & -\rho\psi \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix}.$$

Since

$$k_{ss}^{-1}\lambda_{ss}^{-1/\sigma} = \frac{c_{ss}}{k_{ss}} = Az_{ss} = \frac{\beta\rho}{\beta - 1},$$

the characteristic roots are solutions to

$$\begin{vmatrix} \rho\psi\beta\gamma - R & \rho\psi\beta(1 + 1/\sigma\psi)/(\beta - 1) \\ \rho\psi(\gamma - 1) & -\rho\psi - R \end{vmatrix} = 0.$$

One root is positive and one negative, so the steady state is stable. We are interested in the negative root only; call it R_1. The associated eigenvector, call it $v_1 = (v_{11}, v_{12})$, satisfies

$$\frac{v_{11}}{v_{12}} = \frac{\rho\psi + R_1}{\rho\psi(\gamma - 1)}.$$

Since $\rho\psi + R_1 < 0$ and $\rho\psi(\gamma - 1) > 0$, it follows that v_{11} and v_{12} have opposite signs. Thus, the costate falls as capital grows toward the steady state.

Total pollution is

$$x = Akz^{\beta} = Ak(m\lambda k^{1-\gamma})^{\beta\psi} = A\left((m\lambda)^{\beta} k^{\beta - 1}\right)^{\psi}.$$

Since $\psi > 0$, total pollution falls along the optimal path if and only if

$$\frac{1}{\psi}\frac{\dot{x}}{x} = (\beta - 1)\frac{\dot{k}}{k} + \beta\frac{\dot{\lambda}}{\lambda} < 0.$$

This inequality holds for $k < k_{ss}$ near the steady state if and only if

$$(\beta - 1)\frac{v_{11}}{v_{12}} + \beta > 0,$$

which in turn holds if and only if $\sigma > (1 - 1/\beta)$.

Section 2. For the model with exogenous technical change, the optimal path satisfies

$$c = \lambda^{-1/\sigma},$$

$$z = \begin{cases} 1, & \text{if } \lambda \geq (e^{gt}k^{\alpha})^{\gamma - 1}/m, \\ \left[\lambda m(e^{gt}k^{\alpha})^{1-\gamma}\right]^{\psi}, & \text{if } \lambda < (e^{gt}k^{\alpha})^{\gamma - 1}/m, \end{cases}$$

$$\frac{\dot{k}}{k} = \begin{cases} Ae^{gt}k^{\alpha-1} - \delta - k^{-1}\lambda^{-1/\sigma}, & \text{if } z = 1, \\ A\left[\lambda m(e^{gt}k^{\alpha})^{\gamma(\beta-1)}\right]^{\psi} k^{-1} - \delta - k^{-1}\lambda^{-1/\sigma}, & \text{if } z < 1, \end{cases}$$

$$\frac{\dot{\lambda}}{\lambda} = \begin{cases} \rho + \delta + \alpha(A/\beta)(e^{gt}k^{\alpha})^{\gamma}(\lambda mk)^{-1} - \alpha Ae^{gt}k^{\alpha-1}, & \text{if } z = 1, \\ \rho + \delta - \alpha(\beta - 1)(A/\beta)\left[\lambda m(e^{gt}k^{\alpha})^{\gamma(\beta-1)}\right]^{\psi} k^{-1}, & \text{if } z < 1, \end{cases}$$

where m and ψ are as before.

Along a balanced growth path, capital, output, and consumption grow at a common, constant rate, so the output–capital and consumption–capital ratios,

$$M \equiv \frac{y}{k} = A\left[m\lambda(e^{gt}k^{\alpha})^{\gamma(\beta-1)}\right]^{\psi} k^{-1}, \tag{A.1}$$

and

$$a \equiv \frac{c}{k} = \lambda^{-1/\sigma}k^{-1},$$

are constant. Using these two conditions, we find that along the balanced path

$$g_{\lambda} = [\beta\gamma - 1 - \alpha\gamma(\beta - 1)]g_k - \gamma(\beta - 1)g,$$

and

$$g_{\lambda} = -\sigma g_k,$$

so g_k satisfies (3).

Stability can be checked by linearizing around the steady-state values for M and a. The laws of motion for k and λ in the region where $z < 1$, expressed in terms of M and a, are

$$\frac{\dot{k}}{k} = M - \delta - a, \quad \text{and} \quad \frac{\dot{\lambda}}{\lambda} = \rho + \delta - \omega M, \tag{A.2}$$

where

$$\omega \equiv \alpha\frac{\beta - 1}{\beta} < \alpha.$$

Hence

$$a_{ss} = M_{ss} - \delta - g_k, \quad \text{and} \quad M_{ss} = \frac{\rho + \delta - g_{\lambda}}{\omega}. \tag{A.3}$$

Differentiating (A.1), substituting from (A.2), linearizing around the balanced growth path, and letting

$$p \equiv \ln\left(\frac{M}{M_{ss}}\right) \quad \text{and} \quad q \equiv \ln\left(\frac{a}{a_{ss}}\right),$$

we find that

$$\begin{pmatrix} \dot{p} \\ \dot{q} \end{pmatrix} \approx \begin{pmatrix} (\omega - 1)M_{ss} & -[\psi(\beta - 1)\alpha\gamma - 1]a_{ss} \\ (\omega/\sigma - 1)M_{ss} & a_{ss} \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix}.$$

Using (A.3) and the transversality conditions, we find that $\mathrm{tr}(J) > 0$ and $\det(J) < 0$. Therefore the roots are real and of opposite sign, so the balanced path is locally stable.

Section 3. For the model in which pollution enters utility as a stock, the conditions for a maximum are

$$c^{-\sigma} = \lambda,$$

$$z = \begin{cases} 1, & \text{if } \lambda \geq \mu\beta, \\ (\lambda/\mu\beta)^{1/(\beta-1)}, & \text{if } \lambda < \mu\beta, \end{cases}$$

$$\lambda - \mu\beta z^{\beta-1} \geq 0, \quad \text{with equality if } z < 1,$$

$$\dot{\lambda} = (\rho + \delta)\lambda - \alpha A e^{gt} k^{\alpha-1} z(\lambda - \mu z^{\beta-1}),$$

$$\dot{\mu} = (\rho + \eta)\mu - Bx^{\gamma-1},$$

where λ and μ are the costates for k and x. The sign of the latter has been reversed, so $\mu > 0$.

In the region where $z < 1$, the laws of motion are

$$\frac{\dot{k}}{k} = M(t) - \delta - a(t), \tag{A.4}$$

$$\frac{\dot{\lambda}}{\lambda} = \rho + \delta - \omega M(t),$$

$$\frac{\dot{x}}{x} = N(t) - \eta,$$

$$\frac{\dot{\mu}}{\mu} = \rho + \eta - b(t),$$

where

$$\text{(A.5)} \qquad M \equiv \frac{y}{k} = Ae^{gt}k^{\alpha-1}\left(\frac{\lambda}{\beta\mu}\right)^{1/(\beta-1)},$$

$$b \equiv \frac{Bx^{\gamma-1}}{\mu},$$

$$a \equiv \frac{c}{k} = \lambda^{-1/\sigma}k^{-1},$$

$$N \equiv \frac{yz^{\beta-1}}{x} \equiv \frac{Ae^{gt}k^{\alpha}}{x}\left(\frac{\lambda}{\beta\mu}\right)^{\beta/(\beta-1)}.$$

All these ratios are constant along the balanced path, so the growth rates satisfy

$$g_\lambda - g_\mu = (\beta-1)[(1-\alpha)g_k - g],$$

$$g_\mu = (\gamma-1)g_x,$$

$$g_\lambda = -\sigma g_k,$$

$$g_\lambda - g_\mu = g_x - g_k.$$

Using these facts we obtain the grow rates for capital and for total pollution in (3) and (6).

Differentiating (A.5), using (A.4), taking a linear approximation in the neighborhood of the balanced path, and letting

$$p = \ln\left(\frac{M}{M_{ss}}\right), \quad q = \ln\left(\frac{b}{b_{ss}}\right), \quad r = \ln\left(\frac{a}{a_{ss}}\right), \quad s = \ln\left(\frac{N}{N_{ss}}\right),$$

we find that

$$\begin{pmatrix} \dot p \\ \dot q \\ \dot r \\ \dot s \end{pmatrix} \approx \begin{pmatrix} (\omega-1)M_{ss} & b_{ss}/(\beta-1) & (1-\alpha)a_{ss} & 0 \\ 0 & b_{ss} & 0 & (\gamma-1)N_{ss} \\ (\omega/\sigma-1)M_{ss} & 0 & a_{ss} & 0 \\ 0 & b_{ss}\beta/(\beta-1) & -\alpha a_{ss} & -N_{ss} \end{pmatrix} \begin{pmatrix} p \\ q \\ r \\ s \end{pmatrix}.$$

In addition, summing the laws of motion in (A.4) for k and λ and for x and μ, and defining $\hat\rho \equiv \rho - (1-\sigma)g_k$, we find that

$$(\omega-1)M_{ss} = -a_{ss} + \hat\rho, \quad \text{and} \quad b_{ss} = N_{ss} + \hat\rho.$$

Substituting into the matrix above, we find that the characteristic roots are solutions of $\det(D) = 0$, where

$$D = \begin{pmatrix} a_{ss} - (\hat{\rho} - R) & -b_{ss}/(\beta - 1) & -(1-\alpha)a_{ss} & 0 \\ 0 & -N_{ss} - (\hat{\rho} - R) & 0 & -(\gamma - 1)N_{ss} \\ (\omega/\sigma - 1)M_{ss} & 0 & a_{ss} - R & 0 \\ 0 & b_{ss}\beta/(\beta - 1) & -\alpha a_{ss} & -N_{ss} - R \end{pmatrix}.$$

The terms R and $(\hat{\rho} - R)$ enter symmetrically in this determinant, so the roots come in pairs: if R is a root, then so is $R' = \hat{\rho} - R$. Hence we can factor the roots into a quadratic, so

$$\det(D) = X^2 + X\left[a_{ss}(a_{ss} - \hat{\rho} + H) + N_{ss}(N_{ss} + \hat{\rho} + J)\right]$$
$$+ a_{ss}N_{ss}\left[(a_{ss} - \hat{\rho} + H)(N_{ss} + \hat{\rho} + J) + \frac{\alpha}{\beta(1-\alpha)}HJ\right],$$

where $X = R(\hat{\rho} - R)$, and

$$H \equiv \left(\frac{\omega}{\sigma} - 1\right)(1 - \alpha)M_{ss}, \quad \text{and} \quad J \equiv \frac{\beta(\gamma - 1)}{\beta - 1}b_{ss}.$$

Thus, X solves a quadratic equation of the form $X^2 + BX + C = 0$, and the roots are real if and only if $B^2 - 4C \geq 0$. A sufficient condition for real roots is $\sigma > \alpha$, which clearly holds if $\sigma > 1$. Then, since $B, C > 0$, it follows that the two roots X_i are real and negative, so the four roots R_i are real, and exactly two are negative.

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